COMMUNICATIONS TO THE EDITOR

Optimization of Multistage Branching Systems

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Aris, Nemhauser, and Wilde (*1*) have presented a survey of strategies for optimizing complicated multistage decision processes. I should like to point out some other strategy based on decomposition which is proving useful for certain kinds of multistage decision processes.

Let us consider first a simple multistage decision process (Figure 1) under the following assumptions:

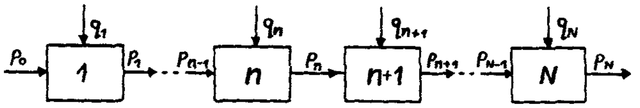

Fig. 1. Simple multistage decision process.

1. For stages 1, . . *i*, . . *n*, the reverse transformation $p_{i-1} = U_i(p_i, q_i)$ and the return function $R_i(q_i, p_i)$ are given.
2. For stages $n + 1$, . . *j*, . . *N*, the direct transformation $p_j = T_j(p_{j-1}, q_j)$ and the return function $P_j(q_j, p_{j-1})$ are given.

Let us optimize (Figure 2) the first part of the process, including stages 1, . . *i*, . . *n*, as if the other did not exist. The optimal policy $q_i^* = q_i^*(p_i)$ for $1 \leq i \leq n$ is obtained by the equations

$$g_i(p_i) = \max_{q_i} \{R_i(q_i, p_i) + g_{i-1}[U_i(p_i, q_i)]\} \quad (1)$$

where

$$g_1(p_1) = \max_{q_1} R_1(q_1, p_1)$$

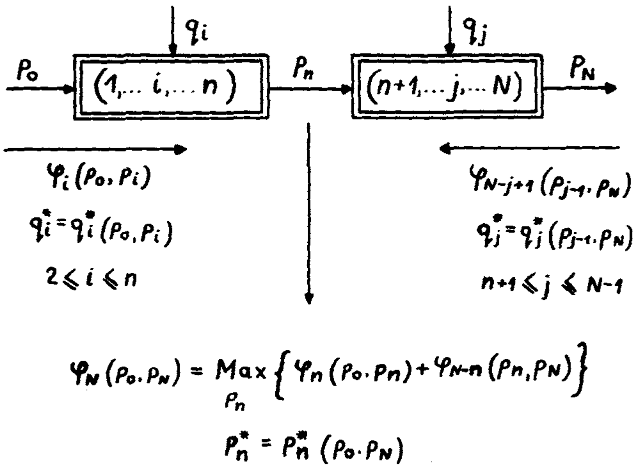

Fig. 2. Decomposition strategy.

The second part of the process, including stages $n + 1$, . . . *j*, . . . *N*, is to be optimized as if the first part did not exist. The optimal policy $q_j^* = q_j^*(p_{j-1})$ for $n + 1 \leq j \leq N$ is obtained by the equation

$$f_j(p_{j-1}) = \max_{q_j} \{P_j(q_j, p_{j-1}) + f_{j+1}[T_j(p_{j-1}, q_j)]\} \quad (2)$$

where

$$f_N(p_{N-1}) = \max_{q_N} P_N(q_N, p_{N-1})$$

The maximum return from the complete process, including stages 1, . . . *N*, is given by

$$\phi = \max_{p_n} \{g_n(p_n) + f_{n+1}(p_n)\} \quad (3)$$

By knowing from (3) the optimum state p_n^*, we can use the before obtained optimum policy functions $q_i^* = q_i^*(p_i)$ for $1 \leq i \leq n$ and $q_j^* = q_j^*(p_{j-1})$ for $n + 1 \leq j \leq N$ to calculate the optimum policy for the whole process.

The proposed strategy is particularly useful for processes with converging and diverging branches (Figure 3). If the stages 1^{I}, . . n^{I}, . . N^{I}, 1^{II} . . ., n^{II}, . . . N^{II}, A, 1, . . . *n* fulfill assumption 1 and the stages $n + 1$, . . *N*, 1^{III}, . . . n^{III}, . . . N^{III}, 1^{IV}, . . . n^{IV}, . . ., N^{IV} fulfill assumption 2, the proposed strategy can be successfully applied in a straightforward fashion, maximizing functions of only one variable, *p* or *q*. The last statement remains true, no matter what the number of converging and diverging branches is.

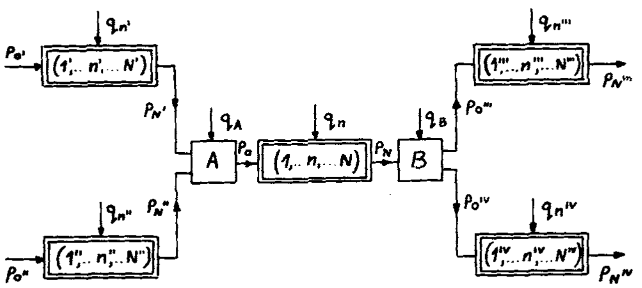

Fig. 3. Multistage decision process with converging and diverging branches.

LITERATURE CITED

1. Aris, Rutherford, G. L. Nemhauser, and D. J. Wilde, *A.I.Ch.E. J.*, **10**, No. 6, 913 (1964).